

Exercise 28

Use a linear approximation (or differentials) to estimate the given number.

$$\cos 29^\circ$$

Solution

Compute the derivative of $y = \cos x$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(\cos x) \\ &= -\sin x\end{aligned}$$

Consequently, the differential of $y = \cos x$ is

$$dy = -\sin x \, dx.$$

In order to estimate $\cos 29^\circ = \cos 29\pi/180$, set $x = \pi/6 \approx 0.52$ and $dx = (29\pi/180) - (\pi/6) \approx 0.017$.

$$dy = \left(-\sin \frac{\pi}{6}\right) \left(\frac{\pi}{6} - \frac{29\pi}{180}\right) = -\frac{\pi}{360} \approx -0.00872665$$

Note that dy here is the vertical distance from the function's actual value at $x = \pi/6$ to the linear approximation's value at $x = 29\pi/180$.

$$\cos 29^\circ \approx \cos \frac{\pi}{6} + \frac{\pi}{360} = \frac{180\sqrt{3} + \pi}{360} \approx 0.874752$$